A Novel Method with Ps Accuracy for Time Interval Measurement

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***Abstract*— High precise time interval measurement is widely used in time synchronization, satellite navigation, aerospace tracking telemetering, laser metering and nuclear electronics. The resolution and accuracy of the current used time counter is 25ps and nearly 100ps, respectively. A new time interval measurement method was put forward, in which, the signal under test is used to trigger a sampler and the phase of the reference sine signal sampled record the time stamp of the test signal. The sampled reference sine signal's phase is estimated by interpolated FFT and 1ps resolution and accuracy can be achieved theoretically. The test experiment of prototype instrument show 10ps accuracy has been achieved. If the hardware prototype is improved to reduce trigger jitter and sampling clock jitter, the measurement instrument may reach 1ps accuracy. The measurement system is used not only time interval measurement, but also for period measurement and modulation domain analysis.**

I. INTRODUCTION

High accuracy time interval measurement instrument has become a key technique in many applications, such as precise time keeping, high accuracy time transfer service, satellite navigation, laser metering and nuclear electronics [1]. Nowadays, the leading time interval counter is SR 620 [2], CNT 90 [3] and Agilent 53132(53131) [4] etc., whose specification is listed as follows:

TABLE I. TABLE 1: THE SPECIFICATIONS OF THE COMMONLY USED TIME COUNTER

Time Counter	Resolution (ps)	Accuracy (ns)	Measurement Rate(S/s)	Frequency Range
SR 620	25	0.1	1K	300MHz
CNT 90	100	0.5	2K	300MHz
Agilent 53132	500	1.0~2.0	200	255MHz
Agilent 53131	150	0.5~1.0	200	255MHz

II. BASIC THEORY

Usually, the measurement method of time counter is to count the pulse number between the start and stop threshold, According to the measured parameter, it can be classified two classes: using the test signal as start and stop pulse to measure period and using the reference signal as start and stop pulse to measure frequency. Since the period and frequency is reciprocal, it's easy to make conversion between them. [5] To improve measurement accuracy, if the frequency of test signal is high, the frequency measurement is used; on the contrast, the period measurement is used. The inherent of these two methods is to measure time interval between two trigger points (usually, zero-crossing point). The usual time interval measurement method is high speed counter but some interpolation method by enlarge error to improve accuracy. The analog interpolation method is used in SR 620 to achieve 25ps resolution and 100ps accuracy. [6]

If we can record the trigger point of test signal and measure the time interval between start and stop trigger point precisely, then we can design a high accuracy time interval counter. Thus, if the test signal is used to control a sinusoid signal sampling system, then the trigger point can be recorded in the phase of the sampled signal. By estimating the phase of the sampled signal, we can get precise time interval measurement. Supposing the sampled signal is 10MHz sinusoid signal with 100MS/s and 10 bits ADC and the accuracy of phase estimation is 0.36^0, then the time measurement accuracy can reach $1/10^7 \times (0.36^0/360^0)$=100ps; The resolution can be calculated as $1/10^8/1024s \approx 10ps$. Thus, time interval measurement is converted to carrier phase measurement. The principle of the new time interval measurement method (for convince, the new method is named sampling measurement method) is as follows:

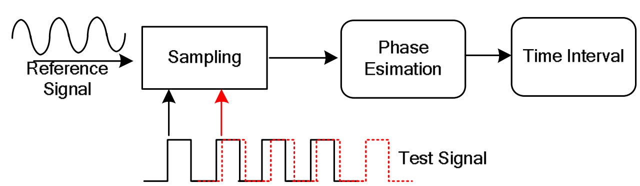

Figure 1. High accuracy time interval measurement block digram

Since time stamp of the trigger point (usually, zero-crossing points) is recorded, the novel time interval measurement method is used not only for time interval but also period, frequency and even modulation domain analyzer.

III. PERFORMANCE ANALYSIS

The important four specification of time interval measurement system are resolution, accuracy, measurement rate and input signal frequency range, which will be analyzed as follows.

The factor affecting the measurement rate contains: test signal frequency, data length of phase estimation, speed of estimation algorithm. If the frequency of test signal is low, the rate is determined by test signal frequency of pulse rate. For example, if the test signal is 1PPS, the measurement rate can't exceed 1M/s. If the frequency of test signal is high enough to exceed the maxim speed of phase estimation processing, then the measurement rate is mainly determined by the data processing. If the data length is 1us and the phase estimation can accomplished real time, then the maxim measurement rate can reach 1M/s.

As usually, the input test signal frequency range is determined by the speed of the font end comparator. For the novel sampling measurement method, the frequency range is mainly determined by the bandwidth of trigger module of the sampling system. According to the specification of current sampling card, the frequency range of the test signal can exceed 300MHz.

The resolution is determined by quantization bits and sampling rate. Assuming the sampling rate is f_s, sampling interval is T_s, the quantization bits is M, then the resolution is $T/2^{\wedge M}$. Suppose f_s is 100MS/s, M is 10, the resolution can reach $1/10^8/1024$s ≈10ps. If the quantization bits 14, then the resolution is better than 1ps ($1/10^8/2^{14}$s≈ 0.6ps).

The measurement accuracy is affected by the quantization bits number, sampling rate, data length, phase (time) estimation algorithm and reference signal's stability. Increasing the bits number of ADC can improve the resolution and get higher SNR and accuracy. The relation between SNR and quantization bits number is shown in the following equation: [7]

$$SNR = 6.02B + 1.76 \tag{1}$$

Where B is the bit number of analog digital conversion (ADC) and the unit of above equation is dB.

The accuracy of phase (time) estimation is concerned with the SNR and data points number N, will be discussed in section 3.1.

The reference signal is sampled to record the trigger point of the test signal whose stability will affect the sampling measurement system. The influence is similar with the traditional time interval counter, which has been analyzed in [4].

A. phase (time) estimation algorithm

The phase (time) parameter estimation of sinusoid signal is a classic problem. There're many algorithm to solve this problem, which can been classified into there types: time domain algorithm, frequency domain algorithm and time-frequency algorithm. The optimal MLE algorithm of parameter estimation for sinusoid signal in AWGN has been studied by Rife[8] ,whose deviation reach the Cramer-Rao bounds. Although the MLE algorithm is optimal, its complexity is two heavy for real time processing. The algorithm with DFT has clearly physical concept and can be implemented by FFT algorithm to achieve good real time property. The DFT can be used detractively to estimate the phase and frequency when the demand accuracy is low.[9] For high accuracy phase estimation, the interpolated DFT can be used. The kernel of interpolated DFT is estimating the bias δ between the maxim large amplitude spectrum line and the real frequency line and using the bias δ to revise the phase and frequency at the maxim amplitude spectrum line. The commonly used interpolated FFT algorithm are: Rife-Jane algorithm [10][11][12], Quinn algorithm[13][14] etc. The amplitude ration of the second large spectrum and the first large is used to estimate the bias in Rife-Jane algorithm; In the Quinn algorithm the real part of the ratio between the second large spectrum and the first one. Since the ratio in Quinn contains phase information, it avoids the direction mistake when the bias is very little; accordingly the computation increases.

The Rife-Jane interpolated DFT algorithm is used to estimate phase (time) because of such reasons: 1) the reference signal is nearly pure signal with very high *SNR* (usually more than 50dB) so the AM noise has little effect on Rife-Jane algorithm; 2) The frequency of the reference signal is known and certain so we can choose proper sampling frequency to ensure the bias δ near to 0.5 to improve estimation algorithm's accuracy, even reach the Cramer-Rao bounds ($1.015\sigma_{CRB}^2$); 3) The Rife-Jane algorithm is concise and can be implemented real time easily.

For the integrality, the basic theory and error of Rife-Jane algorithm will be introduced concisely. Suppose the sampled reference signal is $x(kT_s)$ as follows:

$$x(kT_s) = s(kT_s) + n(kT_s) = A\sin(2\pi f kT_s + \varphi) + n(kT_s)\ , \quad \kappa = 0,\ 1, \dots, N-1 \tag{2}$$

where A, f, φ is the signal amplitude, frequency and phase, T_s is sampling interval, $n(kT_s)$ is zero mean Gaussian whit noise with variance σ^2. Since the sampled reference is nearly pure sinusoid l, the noise is mainly arise from quantization.

After N points DFT of $s(kT_s)$, the discrete spectrum is as follows:

$$\begin{aligned} S(K) &= S_p(K) + S_n(K) \\ &= -0.5jA\begin{bmatrix} e^{j(a(\lambda-K)+\varphi)}\dfrac{\sin\pi(\lambda-K)}{\sin\pi(\lambda-K)/N} \\ -e^{-j(a(\lambda+K)+\varphi)}\dfrac{\sin\pi(\lambda+K)}{\sin\pi(\lambda+K)/N} \end{bmatrix} \end{aligned} \tag{3}$$

where $\lambda = f/f_0$, $f_0 = 1/(NT_s)$, is frequency resolution, $a = \pi(N-1)/N$.

Considering the symmetry of $S(k)$, its positive part $S_p(K)$ contains all the information and the negative part can be omitted. Suppose when $K=L$, $S_p(K)$ is maxim, then the second large spectrum line may locate as $K=L+i$ (i=1 or -1).

According to (3), the amplitude of the two spectrum line can be calculated as

$$\left|S_p(L)\right| = 0.5A\frac{\left|\sin(\pi\delta)\right|}{\sin(\pi\delta/N)} \approx 0.5A\frac{\left|\sin(\pi\delta)\right|}{\pi\delta/N} \tag{4}$$

$$\left|S_p(L+i)\right| = 0.5A\frac{\left|\sin\pi(1-i\delta)\right|}{\sin\pi(1-i\delta)/N} \approx 0.5A\frac{\left|\sin\pi(1-i\delta)\right|}{\pi(1-i\delta)/N} \tag{5}$$

where $\delta=\lambda-L$, respect the bias between f and the frequency at the maxim large amplitude, whose range is [-0.5,0.5]. It can be found that if the bias δ can be estimated precisely, the phase, frequency of the test signal can be obtained.

When N is larger than 1024 and λ is not less than 20, the error in (4) and (5) is no more than 0.015%[11]. From (4) and (5), the bias δ can be solved as

$$\delta = \frac{i\left|S_p(L)\right|}{\left|S_p(L+i)\right| + \left|S_p(L)\right|} \tag{6}$$

Thus, the phase estimation value is

$$\tilde{\varphi} = phase(S_p(L)) - a\delta + \frac{\pi}{2} \tag{7}$$

The accuracy of phase estimation depends on the accuracy of the bias δ. The sinusoid parameter estimation simulation using Rife-Jain in [11] and [12] showed that when N is 1024 or even more, the RMS error of δ is no more than 2×10^{-4}. In fact, the value of the bias has an effect on its estimation accuracy. The standard deviation the bias is given in [15] as follows:

$$std(\tilde{\delta}) = \begin{cases} \sqrt{\dfrac{2}{N\bullet SNR}} & \delta=0 \\ \sqrt{\dfrac{(1-|\delta|)^2\left[(1-|\delta|)^2+\delta^2\right]}{N\bullet SNR\bullet \sin c^2(\delta)} + 2\delta^2 erfc\left[\dfrac{|\delta\sin\pi\delta|}{\pi(1-\delta^2)}\sqrt{N\bullet SNR}\right]} & otherwise \end{cases} \tag{8}$$

In the second item of the above equation is the direction mistake error when the bias δ is very small. Supposing the DFT point number N is 1024, SNR is 50dB, when δ vary in the range [-0.5,-0.5], the standard deviation of δ is shown in Figure 2 and Figure 3.

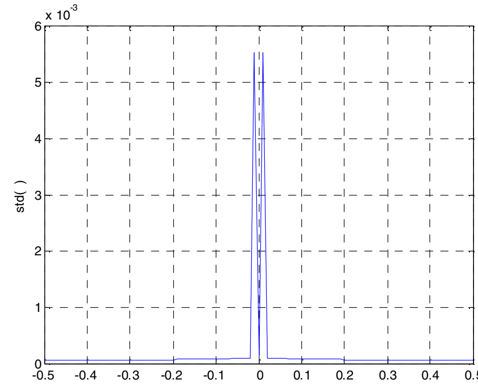

Figure 2. The stand deviation of δ estimation

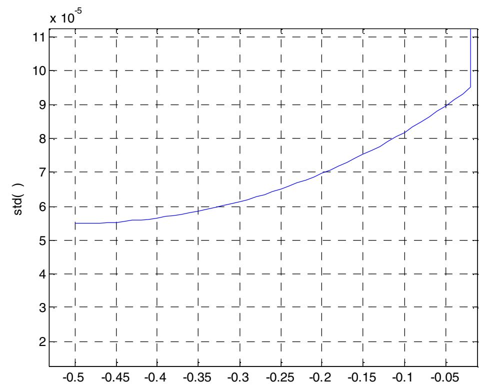

Figure 3. The stand deviation of δ estimation (enlarged partly)

It is clear that when $|\delta|$ is less than 0.02, the direction mistake may increase the estimation error greatly. As long as $|\delta|$ is larger than 0.02, the direction mistake error can be omitted and the standard deviation is mainly the first item of equation (8). It also can be found that when δ is 0.5, the standard deviation is least, equal to 5.49e-5. The Cramer-Rao bounds of frequency estimation is given in [8], from which the Cramer-Rao bounds of bias can be calculated as follows:

$$\sigma_{CR}(\tilde{\delta}) = \frac{\sigma_{CR}(\tilde{f})}{f_0} = \sqrt{\frac{3}{T^2\bullet N\bullet\pi^2\bullet SNR\bullet f_0^2}} = \sqrt{\frac{3}{N\bullet\pi^2\bullet SNR}} \tag{9}$$

With the above equations, the Cramer-Rao bounds of δ can be calculated, equaling to 5.45e-05. When δ is 0.5, the standard deviation of the bias estimated by the Rife-Jane algorithm is 1.0073 times the Cramer-Rao bounds.

From (7) and (8), the standard deviation of phase and time interval estimation can be calculated as

$$std(\tilde{t}) = \frac{std(\tilde{\varphi})}{2\pi f_{ref}} = \frac{std(\tilde{\delta})}{2\pi f_{ref}} \tag{10}$$

For example, when δ is 0.5, the standard deviation of δ is 5.49e-5, then the standard deviation of phase is 5.49e-5 $\times$ a=5.49e-5 $\times$ $(N-1)/N \times \pi$ rad, about 5.49e-5/2 period of reference signal. If the reference signal is 10MHz, whose period is 100ns, then the standard deviation of time interval estimation is 2.745e-5 times 100ns, i.e. 2.745ps.

B. *ambiguity resolving*

If the frequency of the reference signal is f_{ref} Hz, then there's K$\times 1/f_{ref}$ ambiguity in time interval measurement. And the higher frequency means the more frequent ambiguity. Here, we give some method to resolve the ambiguity.

- Aided with a counter to measure the period number of reference signal. The counter can be implanted easily. If its accuracy is no more than the 1/2 of the period, it can work well. Under this condition, the sampling method presented in this paper can be treated as a new "interpolation" method which is more accurate than the traditional interpolation method [6].
- Using more than one reference signal. If two or three reference signal is used to be reference signal, the ambiguity can be solved using the ambiguity resolving method in GPS carrier ranging and multiple tone ranging system. The disadvantage is increasing the sampling system cost and data processing burden.
- Using spread spectrum signal as reference signal like pseudo rang measurement in GPS receiver. When the carrier to noise density ratio is 45dBHz, the standard deviation of pseudo range can reach 0.1ns. Here the carrier to noise density ratio can be very high,, and it's easy to reach 1ps measurement accuracy. The disadvantage is also increasing system cost.

As a summary, the first method is a effective method to resolve ambiguity, which only need an additional counter. The counter is easily implanted because its accuracy and system clock is low.

IV. EXPERIMENT RESULT

A prototype is implemented based the sampling measurement method presented in this paper, whose accuracy has been tested under three conditions:

TABLE II. THREE EXPERIMENT CONDITIONS AND THE CORRESPONDING THEORETICAL ACCURACY (F_{REF}=10MHZ, N=1024)

Sampling rate (MS/s)	100M	25M	100M
Quantization number (bits)	10	14	14
Resolution (ps)	97.66	2.44	0.61
Theoretical *SNR* (dB)	61.96	86.04	86.04
Bias δ	0.40	-0.4	0.40
Theoretical accuracy (ps)	7.1212e-1	4.4520e-2	4.4520e-2

For 10MHz reference signal, 1024 points DFT, the bias, theoretical resolution, SNR and accuracy under three conditions are listed in Table II, which will be used to compare with the experiment result. From Table II, it can be found that the accuracy of all the three conditions is better than 1ps. The phase estimation accuracy is determined by the quantization bits number; the resolution is determined by the sampling rate and quantization bits number. When choosing sampling rate f_s, the Nyquist sampling theorem, resolution and bias should be considered. The resolution and accuracy should be considered when choosing quantization bits number N.

A. *experiment equipment*

The high accuracy sampling measurement prototype is made of two 14-bit 100 MS/s digitizers. The reference signal is Cs atomic clock 10MHz sinusoid which is distributed and used as the two digitizers' reference clock. The test signals are two 1PPS distributed from one clock. The block diagram of the experiment is shown in Figure 4.

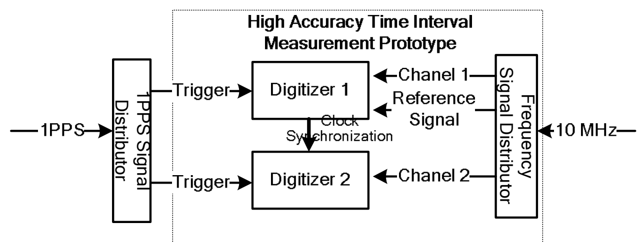

Figure 4. The noise floor of the time interval measurement prototype

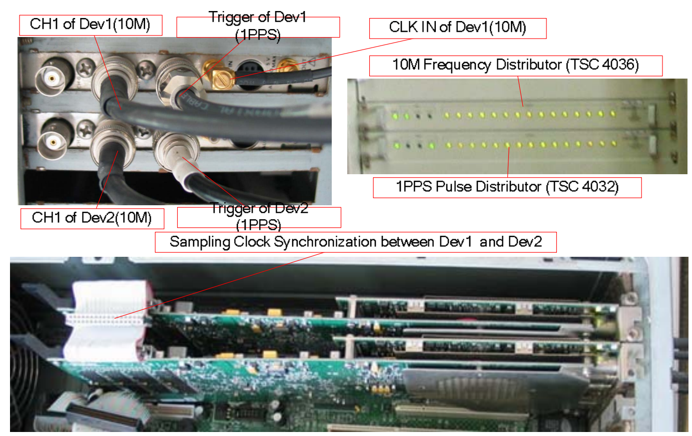

Figure 5. the prototype photo

B. *test results*

The one hundred test results including the phase (time) of channel 1 and channel 2, the interval between two channels with 100MS/s sampling rate and 14 bits quantization are given in Figure 6.

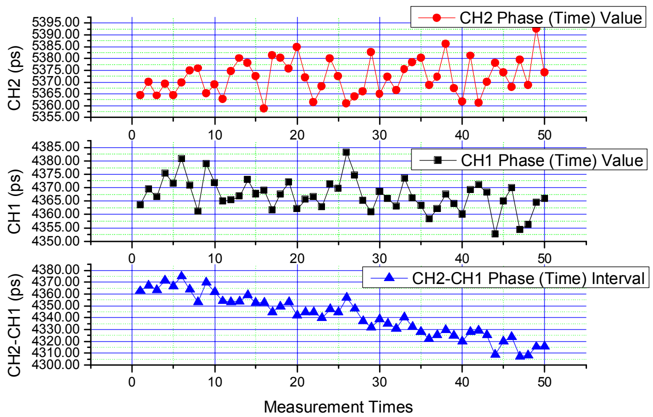

Figure 6. Phase and time interval measurement result (100MS/s sampling rate, 14 bits)

Besides, the detailed results of phase (time) and time interval between two channels under three conditions are listed in Table III:

TABLE III. ACCURACY COMPARISON UNDER THREE CONDITIONS (f_{ref}=10MHz, N=1024)

number	sampling rate (MS/s)	100 MS/s	25 MS/s	100 MS/s
	quantization bits number	10 bits	14 bits	14 bits
Ch1 phase (time) (ps)	standard deviation	8.462	9.2366	7.679
	mean	4368.7000	4373.0687	4366.0252
Ch2 phase (time) (ps)	standard deviation	9.0556	8.3792	7.8868
	mean	5372.6819	5375.3644	5370.2423
Ch2-Ch1 time interval (ps)	standard deviation	10.086	9.706	9.0725
	mean	-1003.9819	-1002.2957	-1004.2171

Analyze the test results in the above table, such conclusion can be obtained:

- The accuracy of signal channel phase (time) measurement is about 10ps, there's no distinct difference between the three conditions. Comparatively speaking, the accuracy of time interval measurement with 100MS/s sampling rate and 14 bits quantization is the best one， which is less than 10ps; The worst accuracy occurred with 100MS/s sampling rate and 10 bits quantization, which exceed 10ps a little.
- The accuracy of all the three conditions is worse than the theoretical accuracy in Table II. It may arise from the trigger jitter and test signal's stability which will affect the signal channel phase (time) measurement accuracy.
- The influence of the test signal stability is concealed in time interval measurement between the two test signals are coherent but the sampling clock synchronization jitter between two digitizers are introduced, so there're no distinct improvement comparing with the signal channels phase (time) measurement.

V. CONCLUSION AND EXPLORATION

From the comparison between theoretical accuracy and experiment results, it can be found that there's no distinct difference between the three conditions because of the measurement noise including trigger jitter and sampling clock jitter. Totally speaking, the accuracy of signal channel phase (time) measurement is about 10ps.

The sampling clock synchronization jitter between two digitizers increases the measurement floor noise. In the further work, the sampling clock synthesizer, analog digital converter (ADC) and data processing FPGA will be integrated in a PCB in order to achieve better accuracy, miniaturization and real time. The accuracy of new designed measurement may reach 1ps.

Besides that, the data processing including interpolated FFT phase (time) estimation and conversion from phase to time interval will be implemented in FPGA. Thus, the data processing can be finished in teal time and the measurement rate is mainly influenced by the data length. For 1024 points FFT, 25MS/s sampling rate, the data acquisition time is 4.0960e-005s, so the maxim measurement rate is 2.4414e+004Hz. Increasing the sampling rate or deducing the FFT point number both can improve measurement rate. From the practical demand, the FFT number N can be reduced to 64, sampling rate f_s is 25MS/s, quantization bits number is 14, the theoretical accuracy is still better than 0.2ps.

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